Lecture 12: Work and Kinetic energy

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Kinetic Energy

For an object with mass m and speed v, the kinetic energy is defined as:

$$\mathbf{K} = K = \frac{1}{2} m v^2$$

Kinetic energy is a **scalar** (it has magnitude but no direction); it is **always a positive** number; and it has SI units of $\mathbf{kg \cdot m^2/ s^2}$ This new combination of the basic SI units is known as the **joule**:

$$1 \text{ joule} = 1 \text{ J} = \mathbf{kg \cdot m^2/ s^2}$$

As we will see, the joule is also the unit of work **W** and potential energy **U**. Other energy units often seen are:

$$1\,\text{erg} = 1\,\frac{\text{g}\cdot\text{cm}^2}{\text{s}^2} = 10^{-7}\,\text{J} \qquad 1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$$

Work

When an object moves while a force is being exerted on it, then **work** is being done on the object by the force. If an object moves through a **displacement d** while a constant **force F** is acting on it, the force does an amount of work equal to

$$\mathbf{W = F \cdot d = F\, d \cos \varphi}$$

where φ is the angle between d and F.

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Work done by a **constant** force:

$$W = F\, d \cos Ø = \vec{F} \cdot \vec{d}$$

Consequences: (ø if the angle between $\vec{F}$ and $\vec{d}$)

when $Ø < 90^o$, W is positive

when $Ø > 90^o$, W is negative

when $Ø = 90^o$, $W = 0$

when F or d is zero, $W = 0$

Work ***done on the object by the force***:

–**Positive work**: object receives energy

–**Negative work**: object loses energy

Work is **energy** transferred to or from an object by means of a force acting on the object.

Formal definition: $\boxed{W = \int \vec{F} \cdot d\vec{s}}$

****Special**** case: Work done by a ***constant*** force:

$$W = (\underbrace{F \cos \theta}) \, d = F\, d \cos \theta$$

Component of $\vec{F}$ in direction of $\vec{d}$

Work done on an object moving with constant velocity?

constant velocity => acceleration = 0 => force = 0

=> work = 0

Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle.

$$\Delta K = K_f - K_i = W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

.... or in other words,

Final kinetic energy = Initial kinetic energy + net Work

$$K_f = \frac{1}{2}mv_f^2 = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net}$$

W_{net} is work done by all forces

Consider 1-D motion.

$$W = \int_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} (m\,a)\,dx = \int_{x_i}^{x_f} m\left(\frac{dv}{dt}\right)dx$$

$$= \int_{x_i}^{x_f} m\left(\frac{dv}{dx}\underbrace{\frac{dx}{dt}}_{v}\right)dx = \int_{x_i}^{x_f} mv\left(\frac{dv}{dx}\right)dx$$

$$= \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}mv^2\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \boxed{= K_f - K_i}$$

(Integral over displacement becomes integral over velocity)

So, **kinetic energy** is mathematically connected to **work**!!